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SHIP RESPONSE TO RANGE ACTION IN HARBOR BASINS

By Basil Wrigley Wilson, Assoc. M. ASCE

WATERWAYS DIVISION

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PAPERS

SHIP RESPONSE TO RANGE ACTION IN HARBOR BASINS

By Basil Wrigley Wilson,[1] Assoc. M. ASCE

Synopsis

The conditions under which a moored ship will respond to sea oscillations in harbor basins are examined on a theoretical basis in this paper. The circumstances in which the ship resonates longitudinally with a periodic surge are deduced and are shown to be dependent primarily on the degree of initial tightness of the moorings, the magnitude of the seiche, and the location of the ship within it. The mass of the ship is of only minor importance in longitudinal motion. Transverse motion is shown to be capable of developing impact forces between ship and quay, which increase approximately as the square of the mass of the ship. Generally, the critical periodicities in both longitudinal and transverse motions are found to lie in the range from 0 min to 2 min. Experimental verification of the theory on models is described, and further evidence supporting the theoretical findings is adduced, indirectly, by induction from an analysis of rope breakages and, directly, by measurement and correlation of the motions of prototype ships with the activating seiches. The importance of tight ropes and of shock-absorbing fenders in helping to circumvent troubles from ship ranging is stressed, but the real solution of undesirable ship motion lies in the subjugation of disturbances that have periodicities of less than 2 min.

1. Introduction

The studies reported in this paper arose as part of the researches into the cause and cure of "range" or surge action in Table Bay Harbor, undertaken at Capetown, Union of South Africa, between 1943 and 1946. The general progress of these researches has been outlined briefly in the contemporary "Annual Reports" of the general manager of the South African Railways and Harbours,

Note.—Written comments are invited for publication; the last discussion should be submitted by May 1, 1951.

[1] Asst. Research Engr., Chf. Civ. Engr's Office, South African Railways and Harbours, Johannesburg, Union of South Africa.

but a more coherent, although necessarily curtailed, account of the phenomenon and its investigation is contained in a paper describing the new graving dock at Capetown.[2] The complete manuscript, which was the basis of this paper, has been filed in the Engineering Societies Library[2a] for reference.

Range action or surge action connotes essentially a motion of water induced by seiches, operating within the confines of a harbor basin. The term "seiche," itself, implies a resonant oscillation of water or "standing wave," sustained by disturbing forces, such as marine or atmospheric wave trains, whose impressed frequency coincides, or closely concurs, with the natural frequency of oscillation of the body of water within the boundaries of the basin. The rhythmic variations of water level at the extremity of Lake Geneva, in Switzerland, were described as seiches more than two hundred years ago.

It usually happens that seiches occur in complex combinations involving both fundamental and higher harmonic modes of oscillation along the principal axes of the basin. The higher frequencies of oscillation are undoubtedly stimulated, for the most part, by visible storm swells, entering the basin, whereas the invisible ground swells of the incoming train of sea waves generally promote the fundamental seiches.

All this is considered to have been adequately established from data collected at Capetown, but it is not within the province of this paper to prove that range action is, in fact, so constituted. Rather, the premise which will be adopted is that the agency responsible for ship movement, when range action occurs, is of the nature of a combination of seiches or standing waves.

The effect of severe range action on shipping berthed alongside solid quays or jetties is always impressive and sometimes alarming. For no apparent reason a ship will describe simultaneous translatory motions in three dimensions within the compass of her mooring ropes. There have been cases, especially with large ships, where this action has been severe and prolonged enough to break all the mooring ropes and to splinter the timber fendering between the ship and the quay. On such occasions the ship's shell plating and bulkheads undoubtedly will also suffer damage of a serious kind.

Early, superficial studies of the range problem at Capetown showed that seiches of about 4-min and 5-min periodicities were very prominent in all the basins, and it was at first assumed that these were the ones responsible for the effects observed. In general, however, the movements of ships were dictated by far more rapid perturbations than these, and, as the researches probed deeper, the significance of the seiches of higher frequency grew more evident. Clear-cut relationships were elusive and were always being occluded by puzzling anomalies of ships not behaving as expected, or otherwise reacting in an unexpected manner. For these reasons, it was very uncertain as to what extent ship disturbances were activated by the strong, transverse seiches with periodicities from 1.7 min to 1.9 min found within the basins, or by their prominent second harmonics. The importance of still higher frequency oscillations was equally uncertain.

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[2] "The Sturrock Graving Dock, Cape Town," by David Eric Paterson, *Journal*, Inst. C. E., October, 1947, pp. 346–348.

[2a] 29 West 39th St., New York 18, N. Y.

Observations of ship motion were made as opportunity offered, but it was soon realized that mere measurements of ship movements could not in themselves provide the answer to this general question unless a sufficient number of ships of all sizes could be observed. Practical difficulties, largely influenced by the unpredictable nature of ship behavior, militated against this statistical approach, and the only satisfactory method of treating the problem appeared to be to examine the relationships theoretically and to check them against such observations as became available.

2. The Spring Analogy of Ship Motion

While observing ship movements in the model of Table Bay Harbor, it became apparent that, under identical conditions, a ship restrained by ropes had a much more violent motion than a ship free to move to the fullest extent in the current of a seiche. The movements in the two cases had an important phase difference, that of the moored ship at times being directly opposed to the movement of the free ship and of the current. It was clear that the effect of the mooring ropes was so important that careful attention would have to be devoted to the nature of their action. The problem, indeed, was at once seen to be analogous to that of a spring-suspended mass, whose point of suspension is subject to a periodic displacement. On this analogy the ship would be represented by the suspended mass, the mooring ropes by the spring, and the seiche current by the periodic disturbing force or displacement.

For every ship, according to her size and the slackness or other characteristics of her mooring ropes, there will exist, on this analogy, a critical period at which she will oscillate dangerously, if there exists also a disturbing impulse of the same periodicity. The disturbing periodicities, of course, are more or less fixed, being the oscillations that develop naturally in the harbor basins according to the size and shape of the basins, and it is only when the critical period of the moored ship is in fairly close agreement with the period of sea oscillation that resonance can take place. When this happens, ropes are likely to be broken; and, if the movement is "off and on," fenders can be crushed. This in essence is the idealized nature of the problem as it was envisaged and as it was subsequently studied.

Notation.—The letter symbols in this paper are defined where they are first introduced, in the text or by the illustrations.

3. The Constraint from Mooring Ropes

In broaching the problem it was necessary to know something of the relationship existing between the retarding pull of a ship's ropes and the longitudinal or lateral displacement of the ship.

Fig. 1 may be assumed to represent a single rope or all the ropes collectively at one end of a moored ship, their lengths being S_0 between A, the rest position of the ship's fair-lead (a rope guide), and B, the bollard on the quay, and their lowest point of sag occurring at C, an intermediate point below the quay level. Assuming for the present that the ropes are of uniform weight per foot run, they will hang in the form of the catenary ACB, the known elements of which (under

equilibrium conditions when the ship is at rest) are the vertical and horizontal distances H, h_0 and l_1, l_2, respectively. If the suspension point A suffers a horizontal displacement AA_1 in the plane of the rope, slack is taken up and the rope assumes the configuration of a segment of a new catenary A_1BC_1, whose hypothetical lowest point of sag, C_1, now lies outside of A_1B. The known elements of the catenary in this case are the length of the rope S and the dimensions H and L.

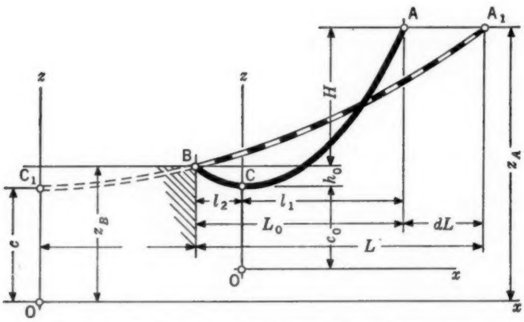

FIG. 1.—GEOMETRY OF MOORING ROPE SUSPENSION

The tensions developing in the rope at any particular stage of displacement of point A are determinable in terms of the density of the rope and the form of the catenary. The latter (in the generalized case when the rope has pulled sufficiently taut to throw C_1 outside of A_1B) may be examined with reference to a coordinate system zOx, the origin of which is taken on the directrix of the catenary A_1BC_1 at a distance c vertically below C_1 (Fig. 1). The general equations of the catenary may then be represented in the forms:

$$z = c \cosh \frac{x}{c} \qquad (1a)$$

and

$$s = c \sinh \frac{x}{c} \qquad (1b)$$

in which s defines the length of the hypothetical rope from point C_1 to any point (x,z) on the catenary. Referring to Fig. 1 and Eq. 1a,

$$z_B + H = c \cosh \frac{x_B + L}{c} \qquad (2a)$$

and

$$z_B = c \cosh \frac{x_B}{c} \qquad (2b)$$

Subtracting Eq. 2b from Eq. 2a,

$$H = c \left(\cosh \frac{x_B + L}{c} - \cosh \frac{x_B}{c} \right) \qquad (3a)$$

Similarly, from Eq. 1b,

$$S = c \left(\sinh \frac{x_B + L}{c} - \sinh \frac{x_B}{c} \right) \qquad (3b)$$

Squaring Eqs. 3, subtracting, and symplifying in terms of standard relationships

for hyperbolic functions,

$$S^2 - H^2 = 2\,c^2\left[\cosh\left(\frac{L}{c}\right) - 1\right]\ldots\ldots\ldots\ldots\ldots(4)$$

Developing the series expansion for cosh L/c,

$$S^2 - H^2 = 2\,c^2\left[\frac{1}{2!}\left(\frac{L}{c}\right)^2 + \frac{1}{4!}\left(\frac{L}{c}\right)^4 + \frac{1}{6!}\left(\frac{L}{c}\right)^6 + \cdots\right]$$

$$= L^2\left[1 + \frac{1}{12}\left(\frac{L}{c}\right)^2 + \frac{1}{360}\left(\frac{L}{c}\right)^4 + \cdots\right]\ldots(5)$$

and, by transposing,

$$\frac{S^2 - (H^2 + L^2)}{L^2} = \frac{1}{12}\left(\frac{L}{c}\right)^2\left[1 + \frac{1}{30}\left(\frac{L}{c}\right)^2 + \frac{1}{1{,}680}\left(\frac{L}{c}\right)^4 + \cdots\right]\ldots(6)$$

On the right-hand side of Eq. 6, each term in the bracket is negligible except the first, 1. Discarding and solving for c^2,

$$c^2 = \frac{L^4}{12}\left[\frac{1}{S^2 - (H^2 + L^2)}\right]\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Since the value of the horizontal tension in a catenary at the lowest point of sag is given by the product $\lambda\, c$, in which λ is the weight per unit length of rope, the horizontal component of rope tension, T_h, at either point A_1 or point B is

$$T_h = \frac{\lambda\, L^2}{\sqrt{12\,[S^2 - (H^2 + L^2)]}}\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

4. Average Mooring Conditions at Capetown

In South African practice ships of the usual sizes are secured fore and aft to the quays by combinations of strops and ship's wires, springs, and manila ropes. According to current terminology in use at Capetown, a "strop" is a sling of heavy coir rope with a steel thimble at one end, through which a ship's wire is threaded and doubled back to the ship, the strop itself being looped over the quay bollard. A "spring," on the other hand, is a loop of coir rope permanently connected to a single steel wire.

A limited number of "backsprings" or side stays is used amidships to increase the holding power; but, for all practical purposes, the moorings of a ship may be considered to consist of two groups (bow and stern) of more or less equally stressed ropes, whose lines of action are oblique to the longitudinal axis of the ship at an angle of about 20°.

The coir strop fulfils in some measure the functions of a mechanical spring by absorbing, gradually, the shock of any suddenly applied load. At Capetown coir strops are of two sizes, 14 in. and 18 in. in circumference, and the common sizes of steel wire used in conjunction with them are 3 in., 3.5 in., and 4 in. in circumference. Coir strops usually have a sling length of 18 ft, and the 18-in. size would weigh about 18 lb per foot of run as against about 2 lb per foot of run

for a single steel wire rope, and 4 lb per ft of run for a double 3.5-in. steel wire rope.

A ship of average size at Capetown (14,200 tons displacement) may be considered to be held with 24 coir ropes of 18-in. size in combination with 18 steel wires of 3.5-in. circumference.

The average parametric dimensions L_0 and H for these moorings are, respectively, 85 ft and 18 ft. The dimensions h_0 and l_2 in Fig. 1 may be assumed, in general, to be, respectively, 2 ft and 10 ft. From knowledge of the dimensions, L_0 and H and l_2 and h_0, the average initial length S_0 of the moorings may be computed to be 88.69 ft.

5. The Elastic Properties of Mooring Ropes

As no reliable information was available in regard to the elastic properties of typical mooring ropes, new and worn samples of coir and steel ropes were subjected to repeated loading tests. To render the results assimilable, mean curves were drawn through the stabilized hysteresis loops of the load-extension diagrams. These gave average relationships between load and extension such

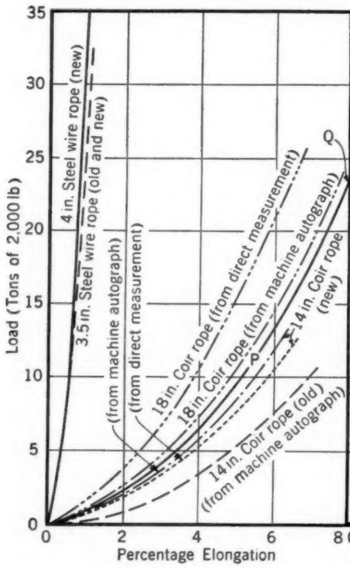

Fig. 2.—Mean Load-Elongation Curves for Coir and Steel Mooring Ropes

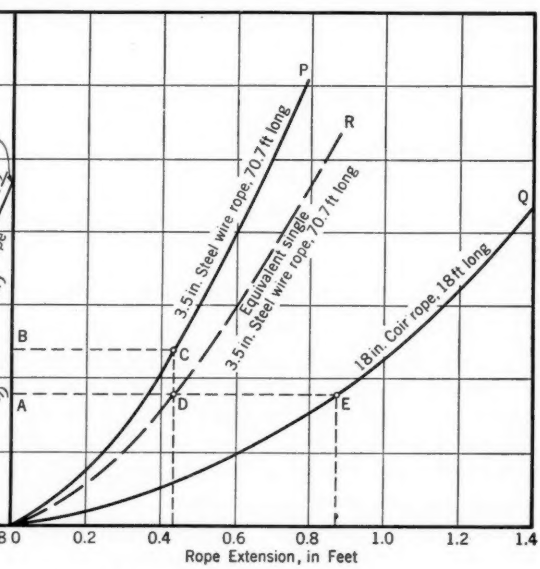

Fig. 3.—Tension-Extension Relationships for Coir-Steel Mooring Combination

as would apply if the ropes were continuously in service. The mean curves for the various ropes tested have been plotted collectively as load-strain diagrams in Fig. 2.

Despite divergencies, a band is indicated within which the load-strain relationship for an average coir rope may be expected to lie. Considering the fact that under average mooring conditions at Capetown a certain number of

14-in. strops are likely to be involved, there is justification for taking, as the final criterion of the elastic properties of an 18-in. coir rope, the arbitrary curve OPQ (Fig. 2), which is centrally disposed in the band. Results for the steel ropes are more consistent and show little variation as between old and new, except in ultimate strength. The difference in shock-cushioning capacity between coir and steel is well portrayed in Fig. 2.

Since the equilibrium length of an average mooring is $S_0 = 88.7$ ft, of which 18 ft comprises 18-in. coir rope and the remaining 70.7 ft comprises 3.5-in. steel rope, the extensions that would occur at various rope tensions in the lengths of the coir and steel may readily be deduced. Curves OCP and OEQ in Fig. 3 demonstrate these relationships, in which curve OEQ corresponds with the curve OPQ in Fig. 2.

An average ship of about 14,200 tons displacement, however, is likely to be tied to its berth with 12/9 coir-steel ropes at both bow and stern, and any pull exerted by the ship on the moorings must, in the idealized case here considered, be distributed uniformly among the 12 coir ropes or among the 9 steel ropes, with the result that an individual steel rope is called upon to carry four thirds of the load imposed on an individual coir rope. It will be necessary, therefore, to adjust the extensions on the steel rope to accord with the fact that the load carried will be four thirds of that borne by the coir rope. Curve ODR, Fig. 3, is the result of such an adjustment.

6. The Relation Between Ship Movement and Rope Tension

As the criterion of the actual tension, T, in the rope at any point, it will suffice to consider merely the horizontal component, T_h. (The error in this assumption can be shown to be insignificant.) Accordingly, Eq. 8 may be converted to present needs. Squaring both sides and transposing, the equation takes the form of a quadratic in L^2, the solution of which is

$$L^2 = \frac{6\,T^2{}_h}{\lambda^2}\left\{\left[+1\,\frac{\lambda^2\,(S^2 - H^2)}{3\,T^2{}_h}\right]^{\frac{1}{2}} - 1\right\}\ldots\ldots\ldots\ldots(9)$$

If λ is computed as the ordinary arithmetical mean of the weights of the coir and steel ropes, its value is found to be 2.4 lb per ft. Since $(S^2 - H^2)$ is likely to have a fairly constant value in the region of 7,700, the term $\lambda^2\,\frac{(S^2 - H^2)}{3\,T^2{}_h}$ will therefore be very small compared with unity for all values of T_h greater than, say, 0.5 ton. Whence, with sufficient approximation,

$$L = \sqrt{S^2 - H^2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

The implication of this Pythagorean relationship is that the tension on the rope is still comparatively insignificant—less than 0.5 ton—when all slack is taken up and the rope becomes completely taut. Measurable tension then develops only when the rope has been pulled into straight-line suspension. It is now seen that the weight of the rope is of no practical consequence.

By selecting arbitrary values of rope tension, T_h, the corresponding extensions, dS (coir + steel), of an equivalent unit mooring can be read from Fig. 3,

and the length of the mooring can be determined as $S = S_0 + dS$. From Eq. 10, L is then simply derived, and the movement of the ship, $dL = L - L_0$, follows at once.

In Fig. 4, a ship is shown, in plan view, moored parallel to a quay at a distance Y_0 from its edge (in the rest position). The moorings AB and A'B' are

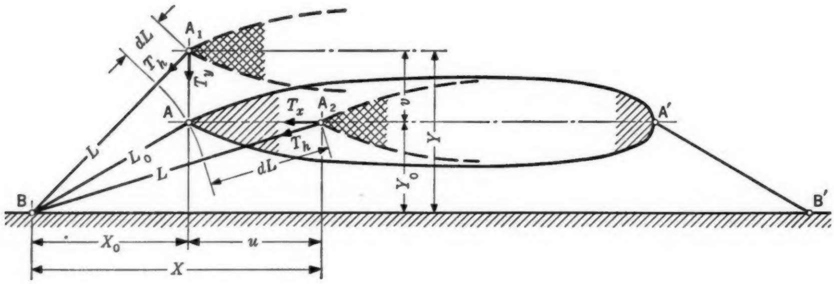

FIG. 4.—GEOMETRY OF LONGITUDINAL AND TRANSVERSE SHIP MOTION

the projections of the ropes on the horizontal plane and correspond with the length L_0 in Fig. 1, when the ship is at rest. It is obvious than any horizontal movement of the ship under the influence of seiches and mooring restraints may be resolved into longitudinal and transverse components relative to the quay, respectively, u and v.

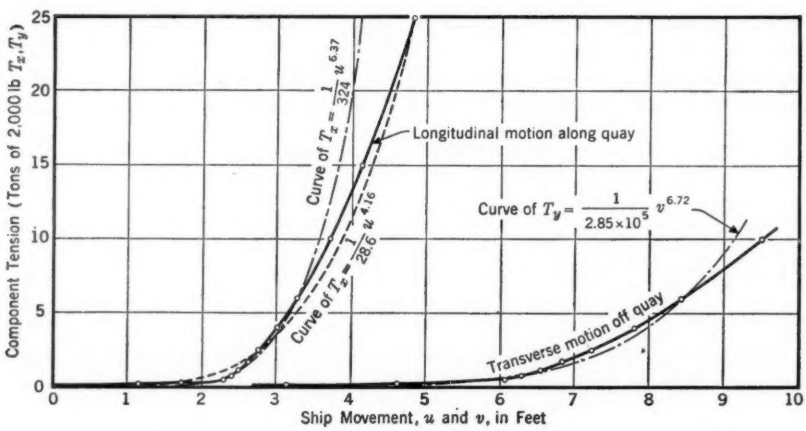

FIG. 5.—RELATIONSHIP BETWEEN ROPE TENSION AND SHIP MOVEMENT

The conversion of dL according to Fig. 4 is represented in Fig. 5. The dimension Y_0 has been assumed equal to 34 ft. Fig. 5 illustrates very clearly that an average ship, moored under the conditions commonly observed at Capetown, is almost completely free to displace longitudinally through a distance of 2.5 ft, and transversely through a distance of 6.5 ft, before any effective rope tension is developed in resisting the movement. The relationships in Fig.

5 conform closely to equations of the following type:

$$T_{x,y} = k\,(u \text{ or } v)^n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

in which k is a constant, and n, a numerical exponent, as may be apparent from the superimposed curves in Fig. 5. Of the two functions shown enveloping the curve for longitudinal motion the one with $1/k = 324$ and $n = 6.37$ is considered to be more appropriate as giving greater accuracy over the range of tensions from 0 ton to 10 tons.

7. Longitudinal Ship Motion Under the Stimulus of a Seiche

In the present treatment of this problem the vertical motion of the ship is neglected and that part of the horizontal motion which is parallel to the quay is considered first. The seiche applicable to these conditions is that which has its node at right angles to the side of the basin or berth along which the ship is moored.

Placing the origin O of a system of coordinates ZOX on the still water surface at one end of the dock (assumed rectangular and of uniform depth), and the x-axis horizontal along the side at which the ship is lying, the z-axis being vertical and positive upward, the horizontal displacement (parallel to the quay) of a water particle at any point (x,z) can be written:[3]

$$\bar{x} = A\,\frac{\cosh q(z+d)}{\sinh q\,d}\,\sin q\,x\,\sin(p\,t+\epsilon) \ldots\ldots\ldots\ldots\ldots (12)$$

in which A is the maximum vertical amplitude (half range) of the seiche; d is the depth of the water in the basin measured from still water level; p is the angular frequency (equal to $2\,\pi/\tau_0$, τ_0 being the period) of the seiche; and q is the nodal frequency for a particular seiche, of value $m\,\pi/D$ (D being the length of the side of the basin and m, an integer defining the nodality of the seiche). In Eq. 12, t has its usual significance of time; and ϵ, of phase angle.

The factor $\frac{\cosh q(z+d)}{\sinh q\,d}$ in Eq. 12 obviously varies from $\frac{1}{\tanh q\,d}$ at water surface to $\frac{1}{\sinh q\,d}$ at sea bottom; and, since the depth d is usually small compared with the dimension D involved in q, the hyperbolic sine and tangent are of the same order, if only the lowest modes of oscillation (such as $m = 1$, 2, and 3) are considered. The horizontal movement of water is thus sensibly the same at all depths and the hyperbolic factor can be replaced by the approximation that $\frac{1}{\tanh q\,d}$ tends to become $\frac{1}{q\,d}$, when the product $q\,d$ is small.

Differentiating Eq. 12 with respect to t, the horizontal velocity of the water at any point is then given by

$$\frac{d\bar{x}}{dt} = V\cos p\,t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

[3] "Hydrodynamics," by Horace Lamb, University Press, Cambridge, England, 1932, p. 356, Article 228.

in which

$$V = \frac{A\,p}{q\,d} \sin q\,x \ldots\ldots (14)$$

The phase angle, ϵ, of the seiche has been discarded for convenience as being unimportant to the argument.

The velocity of the horizontal surge affecting the ship at all depths of its draft at any instant is thus $V \cos p\,t$. The value of V varies, of course, along the length of the ship, but it will be sufficient in most cases to take the value of V corresponding to midposition of the ship and to regard this as representative of water velocity past the ship.

Let the acquired velocity of the ship in the periodic current be v. Then the velocity of the water relative to the ship is $V \cos (p\,t) - v$; and the velocity of the ship relative to the water, $v - V \cos p\,t$.

The force exerted by a stream of water on a submerged cylinder is the product of the mass of the displaced water and the acceleration of the stream in passing the obstruction.[4] It seems justifiable to suppose that the force inducing translation of the ship will be given by the product of the mass of the displaced water (also the mass of the ship M) and the acceleration of the current past the ship.[5] Denoting this force by F,

$$F = M \frac{d}{dt} \Big[V \cos (p\,t) - v \Big] \ldots\ldots (15)$$

The forces opposing the motion of the ship comprise the skin-frictional resistance of the water R, and the retarding pull of the ship's ropes, $\Sigma\,T_x$. The resistance R can be expressed as

$$R = Q\,(v - V \cos p\,t)^2 \ldots\ldots (16)$$

in which Q is a constant depending upon the ship being considered.

The velocity $V \cos p\,t$ of the periodic current of a seiche is never likely to exceed about 1 knot at the most in any deep water harbor. That this is true may be gaged (see subsequently in Eq. 27) from the relation $V = A \sqrt{\frac{g}{d}}$. Under what would be regarded as very severe range action, if $A = 1$ ft and $d = 40$ ft, the surge velocity at a node would not exceed $V = 0.9$ ft per sec. Consequently, the curve of Eq. 16 may be accepted as being reasonably straight over the limited range of small velocities and representable as a straight line with the equation:

$$R = K\,(v - V \cos p\,t) \ldots\ldots (17)$$

The new constant K depends essentially on the size and shape of the ship.

The longitudinal component of the retarding pull of the ship's ropes, ΣT_x, has been shown to be related to the horizontal movement of the ship along the

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[4] "Hydrodynamics," by Horace Lamb, University Press, Cambridge, England, 1932, p. 76, Article 68.

[5] "Theoretical Hydrodynamics," by Louis M. Milne-Thomson, Macmillan Co., London, England, 1938, p. 234, Article 9.22.

quay by the equation of the type $\Sigma T_x = C\, u^n$, in which C is a constant depending on the number and size of the ropes and their condition (Section 6).

It has already been shown that, although there is always tension in the ropes, no matter how slack they may be, such tension is quite negligible compared with the pulls that develop in retarding the movement of the ship. Consequently, when the ship pulls on the stern ropes, the tensions from the bow ropes can be neglected, and vice versa.

By representing ship velocity in terms of the horizontal travel of the ship, Eqs. 15 and 17 may be further developed as follows:

$$F = -M\left(V\, p \sin p\, t + \frac{d^2u}{dt^2}\right) \qquad (18a)$$

and

$$R = K\left(\frac{du}{dt} - V \cos p\, t\right) \qquad (18b)$$

The equation of motion of the ship along the quay under the stimulus of the periodic surge and the restraints of friction and moorings may now be written as follows:

$$M\,\frac{d^2u}{dt^2} = F - R - \Sigma T_x \qquad (19)$$

which reduces (on substitution of the results of Eqs. 11 and 18) to the differential equation:

$$\frac{d^2u}{dt^2} + \frac{K}{2\,M}\,\frac{du}{dt} + \frac{C}{2\,M}\,u^n = \frac{K\,V}{2\,M}\cos p\,t - \frac{V}{2}\,p \sin p\,t \qquad (20)$$

Eq. 20, although representative of a common, natural phenomenon, appears to have no general solution in ordinary transcendental functions. Only in the case when $n = 1$ can the solution be evaluated, such a case representing the damped motion of an ordinary spring-suspended mass under the action of a periodic disturbing force.

8. The Condition of Resonance in Longitudinal Ship Motion

The principal object of the theoretical approach, as outlined in Section 2, was to ascertain the conditions under which ship motion will resonate with the activating periodic surge. The desired information concerns the particular value of the period τ of the seiche which will make the horizontal displacement u, of the ship, a maximum. Any relationship expressing this condition will clearly contain neither of the variables u and t, but only the quantities p (or τ), $\frac{K}{2\,M}$, $\frac{C}{2\,M}$, and V, which are the "foundation stones" of Eq. 20. The elements of the function linking these quantities have been derived by dimensional analysis and the critical periodicity would appear to take the form:

$$\tau = \left(\frac{C}{2\,M}\right)^{-i} V^i \psi\left[\frac{K}{2\,M}\left(\frac{C}{2\,M}\right)^{-i} V^i\right] \qquad (21)$$

in which (to avoid complex typography)

$$i = \frac{1}{1 + n} \quad \text{(22}a\text{)}$$

and

$$j = \frac{1 - n}{1 + n} \quad \text{(22}b\text{)}$$

In the particular instance when $n = 1$, Eqs. 22 reduce to $i = \frac{1}{2}$ and $j = 0$; and Eq. 21 becomes

$$\tau = \left(\frac{C}{2\,M}\right)^{-\frac{1}{2}} \psi \left[\frac{K}{2\,M}\left(\frac{C}{2\,M}\right)^{-\frac{1}{2}}\right] \quad \text{(23)}$$

but, since the known solution for this case, derived from the particular integral of Eq. 20 for $n = 1$, is $\tau = 2\,\pi\left(\frac{C}{2\,M}\right)^{-\frac{1}{2}}$, it follows that the function ψ in Eq. 23 is a constant, of value $2\,\pi$. It is reasonable then to conclude that the function ψ of Eq. 21 is also a constant, of value $2\,\pi$, and the critical resonance condition in the general case thus reduces to

$$\tau = 2\,\pi\left(\frac{C}{2\,M}\right)^{-i} V^j \quad \text{(24)}$$

This result is capable of further development since V is defined in Eq. 14, in which $p = \frac{2\,\pi}{\tau_0}$ and $q = \frac{m\,\pi}{D}$. The periodicity of the seiche τ_0, in the case of a rectangular basin of uniform depth and with vertical walls, such as the Victoria and Duncan docks at Table Bay Harbor, is expressible as[6]

$$\tau_0 = \frac{2\,D}{m(g\,d)^{\frac{1}{2}}} \quad \text{(25)}$$

The maximum velocity V of the horizontal surge of the seiche, at any point x along the side of the basin where the ship is moored, reduces, in consequence (from Eq. 14) to

$$V = A\left(\frac{g}{d}\right)^{\frac{1}{2}} \sin\frac{m\,\pi\,x}{D} \quad \text{(26)}$$

The worst effects from the periodic current obviously occur when $\sin\frac{m\,\pi\,x}{D}$ in Eq. 26 is equal to 1, or when the center of the ship happens to be coincident with the nodal point of the seiche. When the ship is so situated,

$$V = A\left(\frac{g}{d}\right)^{\frac{1}{2}} \quad \text{(27)}$$

If k is the spring constant for an individual mooring rope, as defined in Eq.

[6] "Hydrodynamics," by Horace Lamb, University Press, Cambridge, England, 1932, p. 284, Article 190.

11, and $N/2$ is half the total number of ropes holding the ship, then,

$$C = \frac{N\,k}{2} \qquad (28)$$

Replacing M in terms of W, the displacement tonnage of the ship, and introducing the results of Eqs. 27 and 28, Eq. 24 can be resolved into its final form:

$$\tau = \frac{2\,\pi}{A}\left(\frac{d}{g}\right)^{\frac{1}{2}}\left(\frac{4\,W\,A^2}{N\,k\,d}\right)^{\frac{1}{2}} \qquad (29)$$

The interpretation to be placed on this result is that, if any of the periodicities τ_0 (Eq. 25) of the seiches to which the harbor basin is addicted (in virtue of its dimensions and depth) agrees with or approximates τ in value (Eq. 29)(the resonant periodicity of the ship according to the manner of its mooring), a very dangerous set of circumstances prevails which might easily result in the ship breaking adrift.

As a check on Eq. 29 there remains another possible line of approach to the problem of resonance. The condition of resonance implies equality between the free period of oscillation of the ship-spring system and the impressed periodicity of the seiche, and this, in turn, requires that the maximum travel of the ship, u(max) should equal the maximum amplitude of horizontal movement of the water, $\bar{x}$(max).

The maximum amplitude of horizontal water movement at the node of the seiche is given by Eq. 12 when $\sin\,(p\,t + \epsilon)$ and $\sin\,q\,x$ both equal unity. By the use of Eq. 25, Eq. 12 is further reducible to the form:

$$\bar{x}(\max) = \frac{A\,\tau_0}{2\,\pi}\left(\frac{g}{d}\right)^{\frac{1}{2}} \qquad (30)$$

Replacing $\bar{x}$(max) by u(max) and solving for τ (τ_0 with suffix discarded), the resonance condition for the ship-spring system is expressed as follows:

$$\tau = \frac{2\,\pi}{A}\left(\frac{d}{g}\right)^{\frac{1}{2}} u(\max) \qquad (31)$$

Comparing Eq. 31 with Eq. 29 it would seem that the latter can be valid only if

$$u(\max) = \left(\frac{4\,W\,A^2}{N\,k\,d}\right)^{\frac{1}{2}} \qquad (32)$$

That this identity is actually true is established by the following derivation. The condition of resonance envisaged in formulating Eq. 31 requires that the acceleration of the ship at the extremity of its travel u''(max) shall equal the maximum acceleration of the water mass, $\bar{x}''$(max). Consequently, differentiating Eq. 13 with respect to t, and putting both $\sin\,q\,x$ and $\sin\,(p\,t + \epsilon)$ equal to unity,

$$u''(\max) = \bar{x}''(\max) = \frac{A\,p^2}{q\,d} \qquad (33)$$

The longitudinal component of the pull which this acceleration imposes on the mooring ropes comprises both the inertia force of the ship, $M\, u''(\max)$, and the pressure of the water, $M\, \bar{x}''(\max)$, and is therefore

$$\Sigma(\max)\ T_x = \frac{2\, M\, A\, p^2}{q\, d} \qquad (34a)$$

In terms of Eqs. 11 and 28,

$$\Sigma(\max)\ T_x = \frac{N\, k}{2}\, u^n(\max) \qquad (34b)$$

whence, from Eqs. 34,

$$u^n(\max) = \frac{4\, M\, A\, p^2}{N\, k\, q\, d} \qquad (35)$$

By multiplying both sides of Eq. 35 by $u(\max)$ and eliminating the product $p\, u(\max)$ on the right-hand side in terms of Eq. 31,

$$u^{1+n}(\max) = \frac{4\, M\, A\, p}{N\, k\, q\, d}\left(\frac{g}{d}\right)^{\frac{1}{2}} A \qquad (36)$$

Since $q = \dfrac{m\, \pi}{D}$ and $p = \dfrac{2\, \pi}{\tau}$, the ratio $\dfrac{p}{q}$ in Eq. 36 can be expressed in terms of m, D, and τ, all of which can be further expressed in terms of g and d by the use of Eq. 25. The result of these changes in Eq. 36 establishes the identity of Eq. 32.

Eq. 29 is confirmed, therefore, and exemplifies the penetrating analytical power of dimensional analysis. Eqs. 29 and 31 both represent the resonant condition of ship motion in the longitudinal direction, within the limits of the assumptions made.

Eq. 31 is extremely useful from the point of view of generalization and lends itself to further manipulation. For this purpose it is noted, as in Fig. 5, that $u(\max)$ depends basically on the amount of movement afforded the ship through slackness of its ropes. Thus, the amount, u_0, by which the ship can displace horizontally to draw the catenary of the moorings into a straight line without rope extension (assuming this were possible), is approximately

$$u_0 = (S^2_0 - H^2)^{\frac{1}{2}} - L_0 \qquad (37)$$

With sufficient approximation for most moored ships, the initial length of mooring rope can be expressed as

$$S_0 = \frac{L_0}{2} + \left(\frac{L^2_0}{4} + H^2\right)^{\frac{1}{2}} \qquad (38)$$

This result derives from assuming that, when the ship is at rest, ropes tend to hang in a parabolic catenary, the lowest point of which, at the quay bollard, has a horizontal slope. Substituting Eq. 38 in Eq. 37 and resolving the roots in terms of the binomial theorem—

$$u_0 = \frac{H^2}{2\, L_0} \qquad (39)$$

—a remarkably simple expression. (In the expansions terms involving higher powers than the first of H^2/L^2_0 have been discarded since H is usually less than $0.2\ L_0$ for most conditions of mooring.) If the maximum travel of the ship, $u(\max)$, is greater than u_0 by a factor r, then Eq. 31 becomes

$$\tau = \pi r \left(\frac{d}{g}\right)^{\frac{1}{2}} \frac{H^2}{A\ L_0} \qquad (40)$$

9. Natural Periods of Longitudinal Oscillation for Ships

It has already been shown in Section 6 that the value of the exponent n for moorings, in the avarage case at Table Bay Harbor, is between 6 and 7. If ropes are slacker than average, n will be greater and, if tighter, n will be less than 6 and 7; but even in the case of initially taut ropes n is not likely to be less than about 1.5. It is clear, then, that even comparatively large variations of the quantities inside the second set of parentheses in Eq. 29 will not appreciably affect the $(n + 1)$th root of the total bracketed quantity. This suggests that,

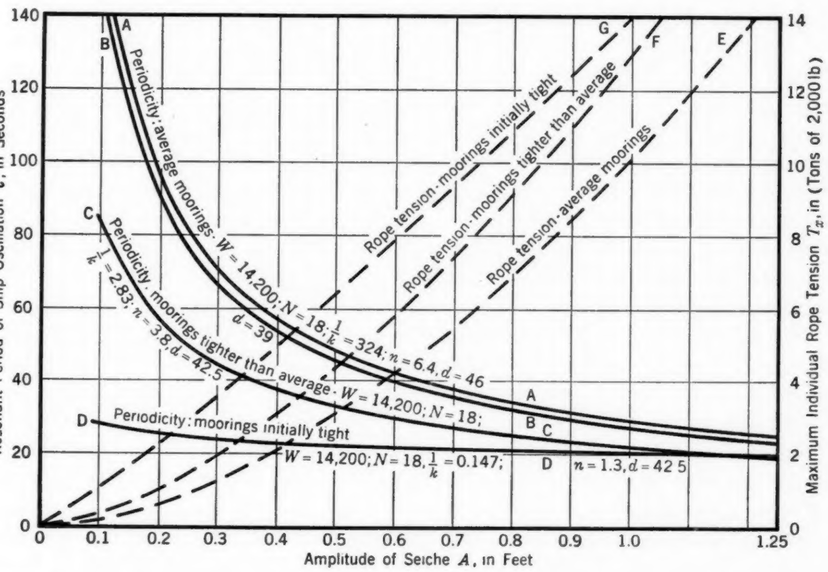


Fig. 6.—Influence of Rope Tightness on the Resonance in Longitudinal Ship Motion

within limits, the mass of the ship and the number of ropes used in mooring it are of minor importance as regards the ship's reaction to the periodic current. By far the most important single factor is the value of n itself. The factor of next importance is the amplitude, A, of the seiche, the periodicity varying roughly in inverse proportion to it.

It is now necessary to determine the free period of oscillation for the average ship ($W = 14{,}200$ tons), whose manner of mooring ($N = 18$; $1/k = 324$; and $n = 6.4$) has been discussed in Sections 5 and 6. The depth of water in Table

Bay Harbor can be assumed to vary from d = 39 ft to d = 46 ft, according to the state of the tide. Thus the variation of free period with amplitude of seiche for these two extremes of tide, as computed from Eq. 29 for the values of the foregoing symbols, is illustrated in Fig. 6 by the curves AA and BB.

These curves are of great interest in so far as they show that seiche periodicities greater than about 2 min are only capable of producing resonance effects in a ship, as normally moored, at very small seiche amplitudes, for which movements would be so feeble as to produce only insignificant rope tensions. This is demonstrated by curve OE, which gives the maximum tension developing in an individual rope at resonance. Data for this curve were computed by Eq. 34a, modified by the use of Eq. 25 to the form:

$$\Sigma(\max)\, T_x = \frac{2\,\pi\, W}{(g\, d)^{\frac{1}{2}}}\, \frac{A}{\tau} \qquad (41)$$

The tensions as calculated by Eq. 41 have been multiplied by a factor of 1.07/9 in conformity with the fact that, in the case treated, the total force is divided among 9 ropes (bow or stern), and that actual rope tensions are approximately 7% greater than their longitudinal components.

The dangerous periodicities for the ship are seen to be in the range from about 1 min down to 25 sec, with resonance at the higher frequencies possible only if the seiche amplitudes are very large. For example, if a seiche of about a 30-sec periodicity and a 1-ft amplitude prevailed in the harbor, then a ship, berthed at a nodal point, could be expected to resonate and impose average maximum tensions on the mooring ropes of about 10 tons each. Allowing for the facts that (1) individual ropes are likely to be overloaded; (2) test results for both new and old coir ropes show ultimate strengths not greatly in excess of 10 tons; and (3) the loading action is cyclic and therefore fatiguing—it will be appreciated that a ship in these circumstances could easily break adrift.

If the ropes of the ship were initially tighter to the extent of making $1/k$ = 2.83 and n = 3.8 the possible range of periodicities at which the ship could resonate dangerously would be further restricted from about 45 sec down to 20 sec (curve CC, Fig. 6). The corresponding rope tensions in this case would be higher (curve OF), and it will be seen that a ship so moored will still be in considerable danger of breaking its ropes under high frequency range action of large magnitude.

If the ropes were initially quite taut for the rest position of the ship ($1/k$ = 0.147 and n = 1.3), the range of resonant periods (curve DD, Fig. 6) would be limited from about 30 sec to 20 sec. A ship so moored would be virtually immune to all seiche disturbances of greater periodicity; but it would still have dangerous response to these high frequency oscillations, as may be judged from the curve OG, which shows the corresponding rope tensions. The mitigating feature here is that most harbors, although usually incapable of preventing ingress of long-period sea disturbances, show greatest efficiency in blocking the penetration of high frequency wave trains. In a well designed harbor, therefore the chances of 20-sec to 30-sec seiches reaching any exceptional amplitude are relatively small, and tight moorings, although no cure-all, are thus a fairly effective antidote to longitudinal ship ranging.

On the other hand, if the ship moorings are slacker than avearge ($1/k$ and n considerably greater, say, than 324 and 6.8, respectively), long-period seiches become increasingly more important in the resonant motion. The relationships for such cases would lie above curve AA, Fig. 6, and the dangerous periodicities would cover a range from 30 sec to 80 sec (or more)—frequencies that have far greater penetrative powers in harbors than ordinary storm swells of 10-sec to 20-sec periodicities, and which are capable therefore of attaining fairly large amplitudes. A poorly tied ship with very slack ropes is thus a ready victim for the onset of severe range action.

The foregoing text treats the case of an average ship of 14,200 tons displacement. It might apply equally well to ships both larger and smaller, that have about the same deck height, since the mass of the ship has relatively little effect

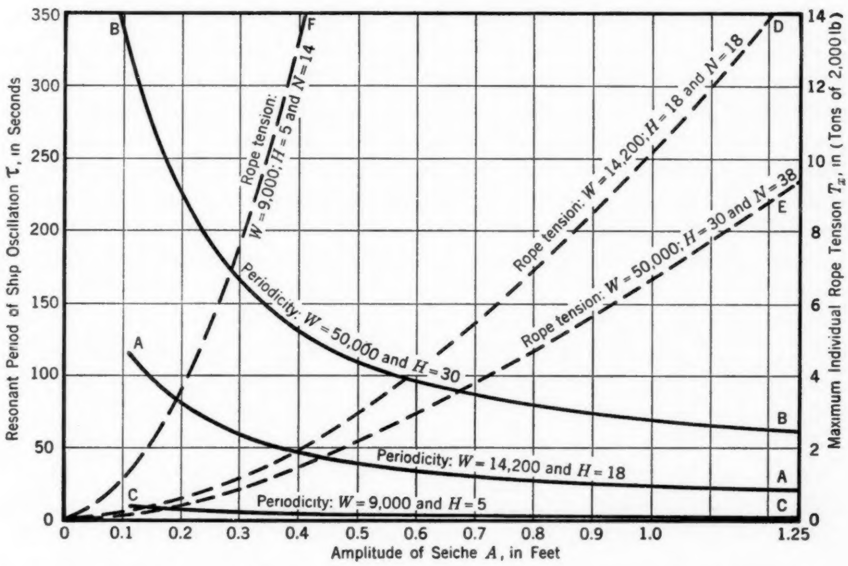


FIG. 7.—INFLUENCE OF SHIP SIZE ON THE RESONANCE IN LONGITUDINAL SHIP MOTION

upon the final result. However, it will be profitable to examine the effects of the size of the ship in so far as it influences the height, H, of the moorings, through the medium of Eq. 40, which expresses the resonance condition in another form.

The criteria are the static parametric dimensions, H and L_0, of the mooring ropes. The distance H is capable of considerable variation according to the size of the ship and the state of its draft; in point of fact, for stern ropes on low lying ships at low tide it can even be negative. The dimension L_0 is less easily affected by ship size and, for convenience, may be considered to remain approximately constant at 85 ft.

The factor r in Eq. 40, being the ratio of u(max) to u_0, will depend on the maximum tension developing in the ropes. An approximate idea of how it

varies with seiche amplitude may be gained from Figs. 5 and 6. As an example, note from Fig. 6 that a seiche amplitude of 0.5 ft will induce a maximum rope tension in average moorings of 3.1 tons. This tension corresponds in Fig. 5 with $u(\max) = 2.85$ ft, whereas u_0 is seen to be about 2 ft. The ratio r, accordingly, for $A = 0.5$ ft, is 1.42. In this way r is found to vary from about 1 to 2 over a range of seiche amplitudes from 0 ft to 1.25 ft. The assumption will be made that the same values of r apply for all values of H—that is, for all sizes of ships, as normally moored.

Taking the mean depth of water as $d = 42.5$ ft, it is then possible to investigate the relationship of Eq. 40 in respect to vessels of three sizes—namely, the average ship of $W = 14{,}200$ tons for which $H = 18$ ft, a leviathan of $W = 50{,}000$ tons for which H might be, say, 30 ft, and a low lying tanker of $W = 9{,}000$ tons for which H might be only 5 ft.

The results of feeding these data to Eq. 40 are set forth in the several curves in Fig. 7. Curve AA is almost identical (as it should be) with the equivalent case, curve AA, in Fig. 6. The large ship of 50,000 tons ($H = 30$ ft), on the other hand, resonates at much lower frequencies (curve BB), and is most dangerous when under the spell of large amplitude seiches of from 2-min to 1-min periodicities. Individual rope tensions, as computed from Eq. 41 (the large ship is assumed to be tied with a total of 38 ropes), are actually lower in this case (curve OE) than for the average ship (curve OD), so that again no importance need be attached to resonance with long-period seiches (2 min to 5 min and more).

The low lying ship of 9,000 tons ($H = 5$ ft) is capable of resonating at only the highest frequencies (less than 10 sec—curve CC), but is nevertheless a dangerous "customer" inasmuch as the tensions developed in its ropes ($N = 14$) are extremely large even at very small seiche amplitudes (curve OF).

The conclusion is now drawn that high riding ships of large tonnage are prone to be influenced more by the longer period seiches in the range from 0 min to 2 min whereas low lying ships respond more to the higher frequencies. The mystery of why certain ships give trouble under quite mild conditions of range, while others successfully ride out much heavier incidences, would now also seem to be explained. The location of the ship, whether at a node or at a loop of a seiche; the manner of its mooring, whether with tight or slack ropes—adequate or inadequate in number; the condition of its draft or the design of its decks, whether high or low in the water; and, above all, the magnitude and periodicity of the seiche disturbance, whether large or small—are the essential factors that determine the reaction of the ship.

10. Transverse Ship Motion Under the Stimulus of a Seiche

Consider next the component of horizontal motion of the ship normal to the quay. Enlarging the system of coordinates previously used to a three-dimensional one, with the x-axis horizontal along the side of the dock at which the ship is lying and the y-axis horizontal at right angles thereto, the z-axis being vertical and positive upward, the horizontal displacement (normal to the quay)

of a water particle at any distance y from the x-axis can be written as follows:

$$\bar{y} = \frac{A}{q' d} \sin q' y \sin (p t + \epsilon) \quad \text{(42)}$$

Eq. 42 is similar to Eq. 12, but refers now to a transverse seiche, whose nodes are at right angles to the y-axis. The symbols have the same designations as before with the difference that q' in this case has the value $m \pi/B$, B being the length or width of the side of the dock parallel to the y-axis.

If the center line of the ship be located at a distance Y_0 from the quay wall, as in Fig. 4, the ship will be subject to a periodic lateral displacement, through the off-and-on movement of the seiche, of

$$\bar{y} = \frac{A B}{m \pi d} \sin \frac{m \pi Y_0}{B} \sin p t \quad \text{(43)}$$

Now, Y_0 will usually be small compared with B, so that, provided the nodality of the seiche is of a low order ($m = 1, 2,$ or 3), the sine involving Y_0 may be replaced by $m \pi Y_0/B$. Hence,

$$\bar{y} = \frac{A Y_0}{d} \sin p t \quad \text{(44)}$$

For the average ship featured in this paper, Y_0 is about 34 ft; and, if the mean water depth be taken as $d = 42.5$ ft, it is clear that even if the amplitude of the seiche, A, is very large, with a value, say, of 1.25 ft, the maximum amplitude of lateral ship movement cannot exceed about 1 ft. It follows then from Fig. 5 that, since the ship must move laterally from the rest position a distance of about 6 ft before any effective rope tension can come into play, no form of resonant transverse motion of the ship is possible. The circumstances too are different from those of longitudinal motion because the only restraints on the movement toward the quay are the virtually inelastic forces in compression of the timber fenders and the ship's shell plating and bulkheads. As often happens, however, the ship may be under the influence of an oblique seiche, especially if located near a corner of the dock, and the longitudinal and transverse components of motion will then synchronize, with the result that the lateral movement of the ship will largely follow the time pattern of the longitudinal movement along the quay.

Most of the severe damage that occurs during range action results from the unsatisfactory cushioning medium to absorb the shock when the ship is impelled toward the quay during the on-movement. It is of interest to estimate the maximum force of impact that a ship may be called upon to absorb when this happens. The worst condition will arise when the ship has a clearance between itself and the fenders exactly equal to the amplitude of the on-movement, $\bar{y}$, for then the impact will occur just as the accelerations of the ship and the water mass reach their peak. These accelerations, given by the second differential coefficient of Eq. 44 with respect to time, are

$$\bar{y}'' = \frac{A Y_0 p^2}{d} \quad \text{(45)}$$

The resisting force $P_y(\max)$ called into play must sustain both the inertia of the ship, $M\bar{y}''$, and the pressure of the water on the ship, also $M\bar{y}''$. Therefore,

$$P_y(\max) = \frac{2\,M\,A\,Y_0\,p^2}{d} \qquad (46)$$

This result may be rendered in different form by the use of Eq. 25, in which B now replaces D, as follows:

$$P_y(\max) = 2\,\pi^2\,m^2\,W\,\frac{Y_0\,A}{B^2} \qquad (47)$$

If the ship also completes a lunge fore or aft at the instant of impact, there will be the additional force of the inward pull of the ship's bow or stern ropes. According to Fig. 4 this force will be Y_0/X of the maximum longitudinal pull, given by Eq. 41. The latter may be somewhat differently stated by substitution from Eq. 25 and, accordingly,

$$\Sigma T_y(\max) = \pi\,m\,W\,\frac{Y_0\,A}{X\,D} \qquad (48)$$

Using now the suffixes x and y to distinguish the amplitudes and nodalities of the longitudinal and transverse seiches, the combination of Eqs. 47 and 48 yields the total force of impact:

$$P(\max) = \pi\,Y_0\,W\left(\frac{2\,\pi\,m^2{}_y\,A_y}{B^2}\right) + \frac{m_x\,A_x}{X\,D} \qquad (49)$$

Eq. 49 reveals at once that it is not the fundamental seiches which have to be feared during range action, but rather their higher harmonics, with nodalities of 2, 3, and higher, because the force increases as the square of the nodality in the principal term. Owing to the approximation made in formulating Eq. 44, Eq. 49 will not be valid for very large values of m; but, in any case, there are limits to the values of m which are applicable, since amplitudes decay with increasing nodality. In any event, the seiche periodicities that will produce the greatest value of $P(\max)$ are between, say, 1 min and 20 sec, for these usually constitute the most active multinodal seiches within a harbor. This deduction is in keeping with the conclusions of Section 9 that the seiches of higher frequency are generally the most dangerous for ships.

If the ship were lying along the shorter side, B, or the y-axis of the dock, the force of impact would be

$$P(\max) = \pi\,Y_0\,W\left(\frac{2\,\pi\,m^2{}_x\,A_x}{D^2} + \frac{m_y\,A_y}{X\,B}\right) \qquad (50)$$

Since B is less than D, it is always easier for a multinodal transverse seiche to maintain itself with larger amplitude than a multitudinal longitudinal seiche of the same periodicity. This fact leads to the general conclusion that Eq. 49 will always give a higher value than Eq. 50, and that damage to ship plating and harbor installations is more likely to occur at berths along the longer side of the dock.

The importance of ship size in lateral impact is a further point of interest, for the force $P(\max)$ increases virtually as the square of the displacement tonnage. This increase occurs because the dimension Y_0 (approximately half the beam of the ship) is itself a function of W. The berthing of very large ships at points in a harbor where the on-movement from transferse seiches is severe should therefore never be allowed unless the berths are equipped with special shock-absorbing fenders.

11. Experiments on Ship Models

An experimental study of the movements of model ships with the object of confirming the theoretical principles evolved in this paper was undertaken at the Range Laboratory at Capetown. Simple apparatus for demonstrating seiches on a model scale, similar to that used by E. Maclagen-Wedderburn,[7] was

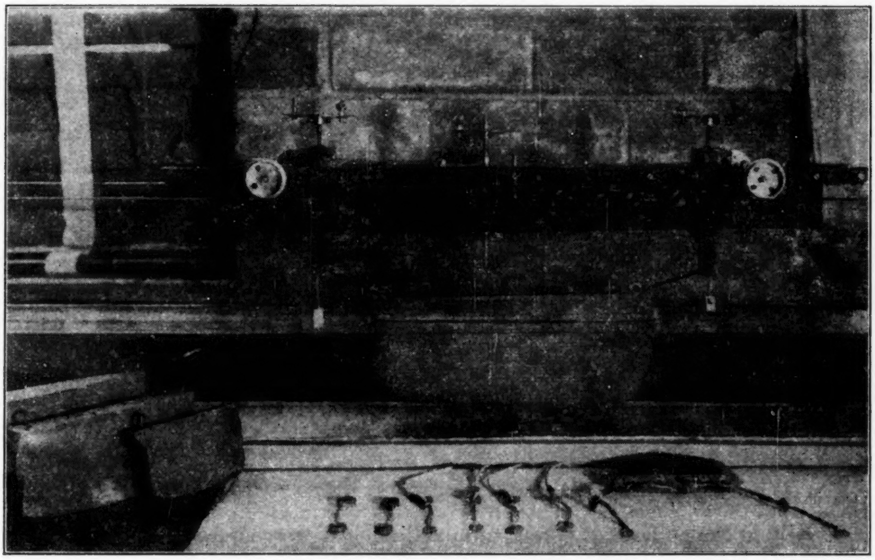


Fig. 8.—Mooring Carriage for Tank Experiments on Model Ships

adapted to good purpose for these experiments. This apparatus took the form of a shallow glass-sided tank 6 ft long, one end of which was partitioned off as a compartment for a displacer, moving vertically in simple harmonic motion. The partition had a narrow opening at the bottom permitting the flux of water necessary to stimulate a seiche in the remaining part of the trough. A second partition (without an opening) could be moved to any position so as to make the effective length of the tank correspond with the period of the displacer, for the production of a resonant seiche.

A mooring carriage (Fig. 8), constructed of toy building parts, was placed across the tops of the glass sides of the tank. The model ships, floating in the

[7] "On the Hydrodynamical Theory of Seiches," by G. Chrystal, *Transactions*, Royal Soc. of Edinburgh, Vol. 41, 1904–1905, Pt. III, pp. 599–649.

water of the trough, were held one at a time with rubber strops to the suspension points below the body of the carriage. These suspension points could be raised or lowered at will, or brought together or farther apart, by the operation of handwheels. By this control the tension in the model ship's ropes could be adjusted to any selected value. The elastic properties of the rubber strops were determined by simple load-extension tests.

The ships, visible in Fig. 8, were modeled in wood with lead inserts to give them the correct buoyancy. The scales used were those of the main harbor model—1/1,200 horizontally and 1/144 vertically. The obvious distortion of the models was of no consequence to the investigation since no attempt was made to translate the model findings to the prototype scale, but, rather, to corroborate theory by a direct application of principles to the model conditions.

Before experiments were made, the critical resonant periodicities at which the various sizes of model ships would oscillate longitudinally, when placed

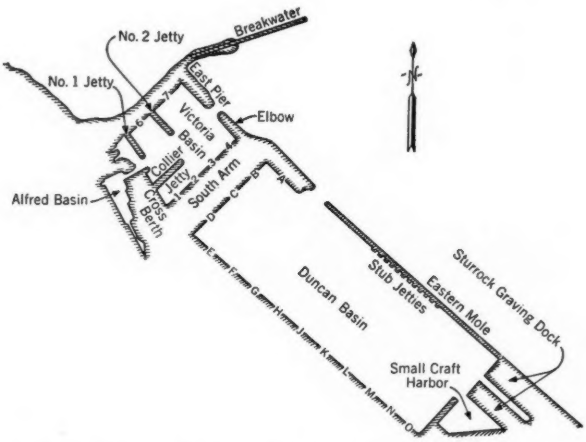


FIG. 9.—TABLE BAY HARBOR, CAPETOWN, UNION OF SOUTH AFRICA

centrally in the nodes of different uninodal seiches, were computed according to the theory evolved in this paper.

In attempting to check the results in the experimental tank, the procedure followed was to take different lengths of the oscillating basin and adjust the speed and amplitude of the displacer so that a uninodal seiche of amplitude 0.7 cm in a depth of water of 9 cm was set up. The model ship, tied tightly by an appropriate pair of ropes to the mooring carriage, was then placed at the node of the seiche and the suspension points moved inward toward each other so as to slack off the ropes to the degree necessary to cause the ship to oscillate most violently in the periodic current of the seiche. The horizontal distance between suspension points could then be measured accurately across the top of the carriage and the value of L_0, producing critical motion, determined. The measured values of L_0 were then checked against those computed.

The voluminous nature of the calculations for the model experiments prevents their incorporation in this paper, but the results of the tests were more

than gratifying and proved that, for the idealized conditions which it treats, the theory is absolutely reliable. One of the most pleasing and impressive demonstrations of the model system was the manner in which it confirmed the importance of tight ropes as a palliative to range action in the mooring of ships.

12. The Statistical Analysis of Rope Breakages at Capetown

The experimental verification of the theoretical approach to the problem of ship ranging permitted the use of the theory in a statistical analysis of the port captain's records of rope breakages at Table Bay Harbor from 1940 to 1946. The length of the paper again precludes the presentation of any details of this application beyond certain findings which have an important bearing on the present theme.

It was found, for instance, that certain berths in Table Bay Harbor are very susceptible to effects from range action, whereas other berths are practically immune. The percentage chances that ships moored at the various berths in the Victoria and Duncan basins (Fig. 9) will part ropes during range action were found on a comparative basis to be as shown in Table 1.

TABLE 1.—Susceptibilities of Berths to Rope Breakage, Table Bay Harbor, Capetown, Union of South Africa

Victoria Basin		Duncan Basin	
Berth	Chance (%)	Berth	Chance (%)
East Pier	11.3	A	21.6
No. 7 Quay	11.1	B	32.7
Outside No. 2 Jetty	20.7	C	18.9
Inside No. 2 Jetty	5.8	D	23.5
North side Collier Jetty	2.0	E	5.3
South side Collier Jetty	9.6	F	6.4
No. 1 South Arm	8.8	G	8.3
No. 2 South Arm	7.2	H	11.0
Nos. 3 and 4 South Arm	17.9	J	9.1
Elbow	20.4	K	6.5
....	....	L	26.3
....	....	M	23.3

In the Duncan Basin, B berth is seen to afford a 1 in 3 chance of rope trouble during range action, and from that point of view quite eclipses any other berth. On the other hand, this berth lies on the node of the second harmonic ($\tau = 0.8 - 1.0$ min) of the transverse oscillation ($\tau = 1.7 - 1.9$ min) for the basin, of which the latter has its node at C berth. It must be inferred, therefore, that the higher frequency seiche must be relatively worse for ships of the size normally accommodated there than for one with a periodicity of about 1.8 min. This inference receives further support from the fact that D berth, coinciding with the other node of the second harmonic, carries also a higher percentage chance of rope breakage than C berth, at the node of the fundamental.

Now the range action model of Table Bay Harbor, besides reproducing the range phenomenon with remarkable accuracy, demonstrated quite conclusively that, although the Victoria Basin could respond to a number of modes of oscillation in the range of periods greater than 4 min, all of these took the form of a general rise and fall of water over the entire area of the dock. In the range of periods of less than 4 min the two most important critical periodicities to which the basin responded covered the bands 2.2 min to 2.5 min and 1.5 min to 1.9 min, of which the latter was much the stronger. Seiches of a periodicity greater than 4 min are obviously incapable of causing rope breakages in the Victoria

Basin and yet, on the whole, the Victoria Basin has fully 70% of the susceptibility of the Duncan Basin to range damage in the form of broken ropes. The only conclusion is that periodicities less than 4 min (and probably less than 2.5 min) are alone responsible for the troubles inherent in range action.

Table 1 records another point of interest in that the inside of No. 2 Jetty and the north side of the Collier Jetty have the lowest susceptibilities of all the berths recorded in the Victoria Basin. Why this should be so may be gleaned from Fig. 9, which shows that No. 2 Jetty and the Collier Jetty form the protecting arms for an inner basin within which the berths in question lie. Since such a system is effective only in damping the penetration of the higher frequency disturbances, the latter are apparently the ones to which ships are most responsive.

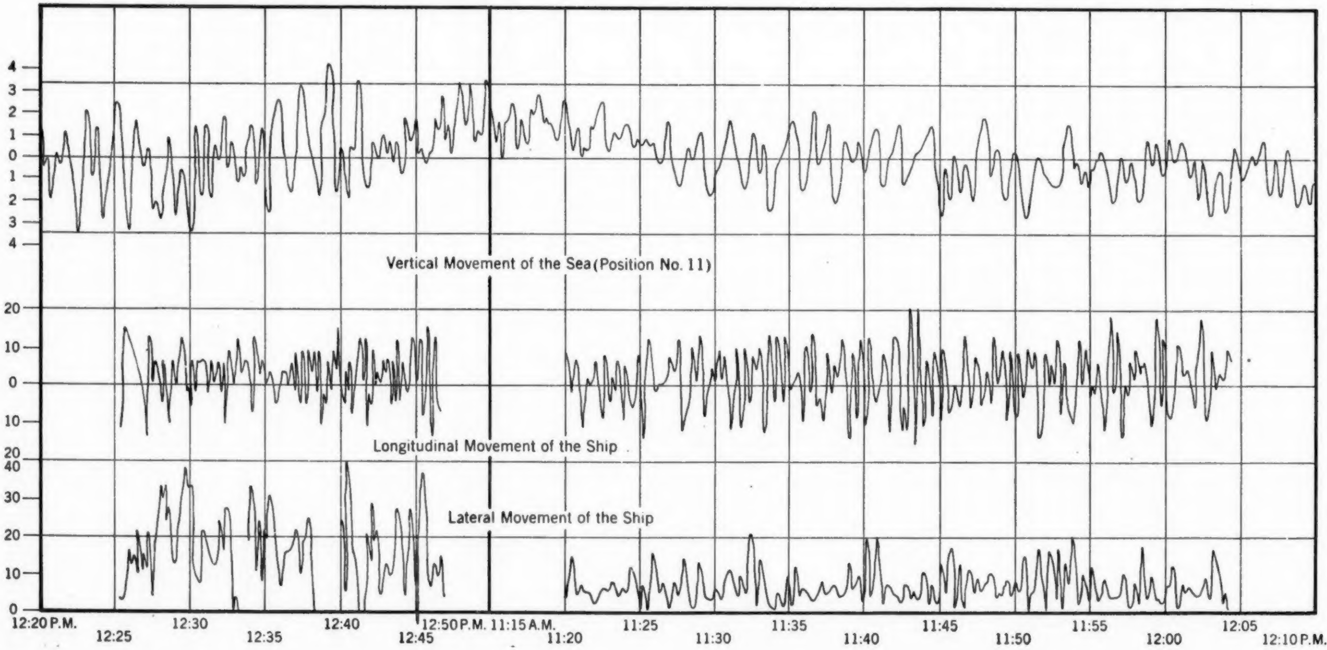
13. Ship Motion by Observation and Measurement

During the course of the researches at Capetown a number of ships of all sizes came under observation. It was the practice, when making measurments of ship motion, to record concurrently the characteristics of the prevailing seiches on "seichometers" or portable tide gages, at points adjoining the berth occupied by the ship. The measurements on the ship comprised simultaneous recordings of the horizontal components of displacement, u and v. Two observers were engaged in this task, one of whom, as the ship reached the extremity of its longitudinal or lateral travel, called out the readings, x or y, on two tapes, one lying on, and parallel to, the edge of the quay, and the other stretched between a point on the ship and the observer's hands. The second observer recorded readings and times to the nearest second. This method, although admittedly subject to error, gave good results in the absence of any complicated self-recording mechanism. No attempt was ever made to record the vertical motions of a ship, since these were considered to be of no particular importance to the investigation.

TABLE 2.—Ships Observed at Capetown, Union of South Africa, June and July, 1945

Ship	Tonnage		Dimensions, in Feet			
	Gross registered	Displacement (when observed)	Length (between perpendiculars)	Beam	Normal loaded draft	Draft (when observed)
Dahlia	5,000	6,400	421	55	25	13
Anna Howard Shaw	8,000	13,400	423	57	27.5	27
H. M. S. Le Tigre	900	2,000	220	35	....	....
Nieuw Amsterdam	36,000	47,700	759	88.3	33.3	33

Typical records for four ships of widely different displacement tonnages (see Table 2), as plotted and correlated with the sea oscillations, are shown in Figs. 10 and 11. The traces in general are complex combinations of harmonic components some of which are easily recognizable, but others of which are

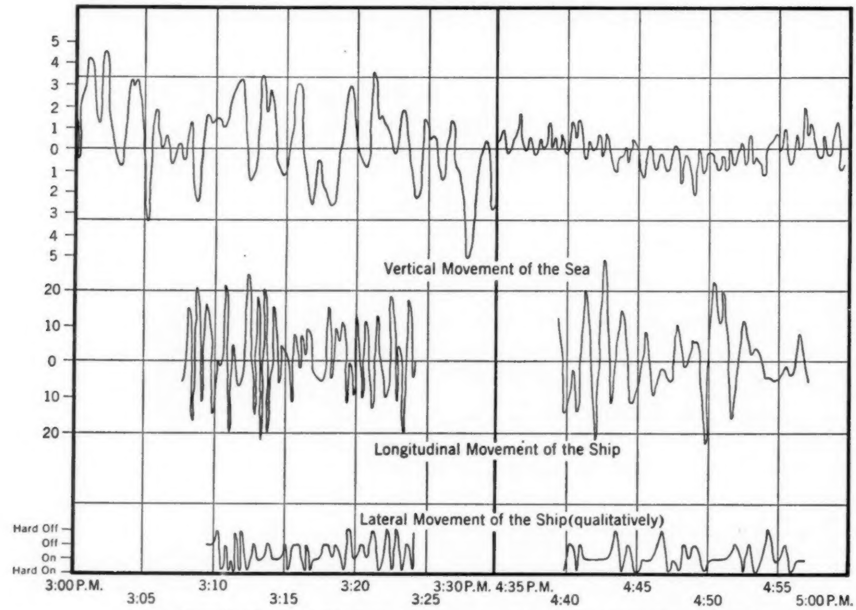


(*a*) *Dahlia* in Berth C, July 7 (*b*) *Anna Howard Shaw* in Berth D, July 31

FIG. 10.—JULY, 1945, OBSERVATIONS BETWEEN SEA OSCILLATIONS AND SHIP MOVEMENTS

inconspicuous. The method[8] of "residuation," introduced by G. Chrystal was used to good advantage in sorting out the various components of motion, and these, in the case of the four ships represented in Figs. 10 and 11, are depicted in Fig. 12, the time base of which is logarithmic for purposes of condensation. The components thus isolated are subject to the errors of analysis, of course, for the residuation process is not infallible, but the writer is inclined to regard them on a statistical basis as being fairly reliable.

One fact emerges at once from Fig. 12—namely, that the dominant periodic components in the ship movements are of the order of 1 min or less, almost irrespective of the size of the ship. The analyses of the water movements



(a) *Le Tigre* in Cross Berth, June 20 (b) *Nieuw Amsterdam* in Berth A, June 21

FIG. 11.—JUNE, 1945, OBSERVATIONS BETWEEN SEA OSCILLATIONS AND SHIP MOVEMENTS

reveal, as expected, the prominent fundamental seiches for the basins and their higher harmonics.

The case of the *Nieuw Amsterdam* (47,700 tons displacement) is of considerable interest in view of the discussion in Section 9. The strongest seiche affecting this ship seemed to be one of 40 sec; yet it chose to resonate with a binodal transverse seiche of about 1-min periodicity, thereby confirming the deduction that large ships tend to resonate at periodicities greater than those of the average size of vessel.

The evidence from Figs. 10, 11, and 12, and, indeed, from all the measurements made on a wide variety of other ships, is conducive to the conclusions drawn previously.

[8] "Investigation of Seiches of Loch Earn by the Scottish Loch Survey," by G. Chrystal, *Transactions*, Royal Soc. of Edinburgh, Vol. 45, 1906, Pt. II, pp. 382–387.

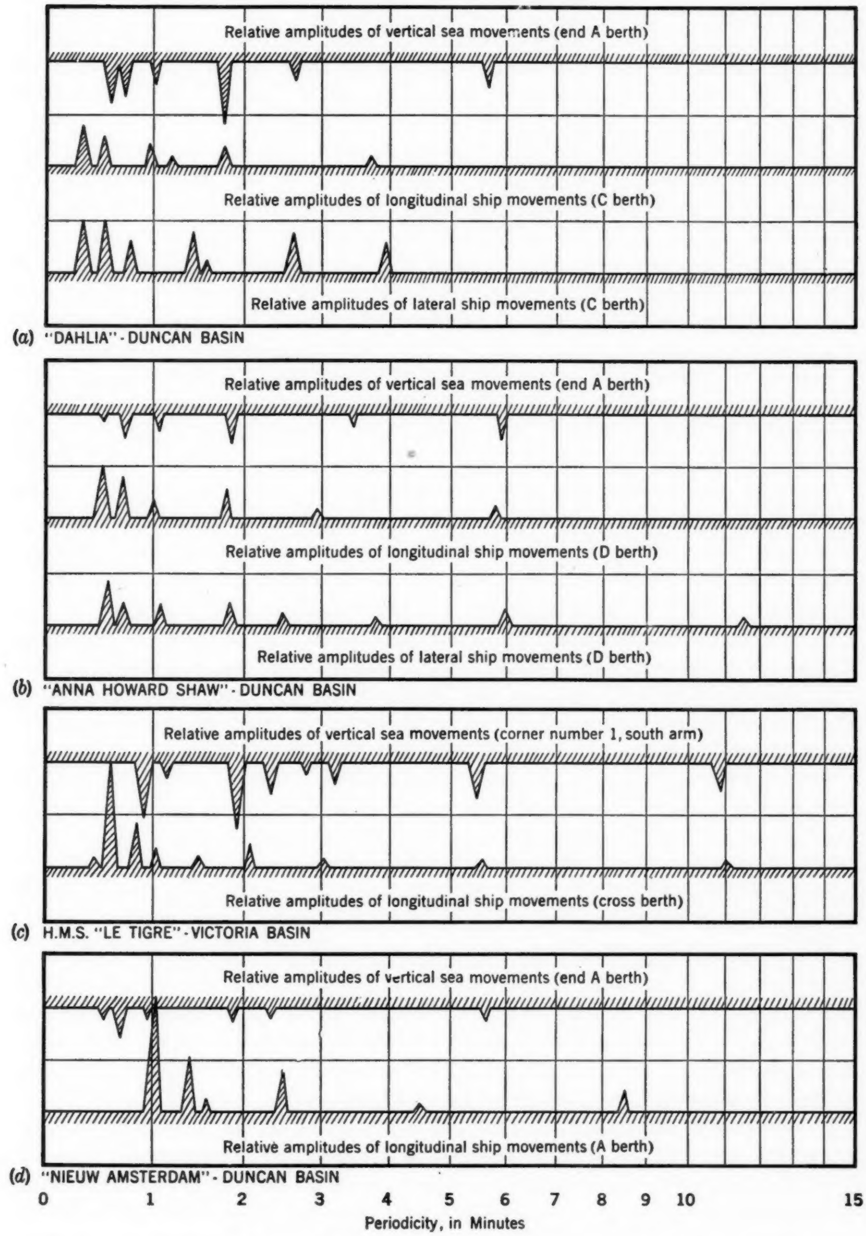


FIG. 12.—CORRELATION BETWEEN SHIP MOVEMENTS AND SEA OSCILLATIONS (PERIODICITIES AND AMPLITUDES OF APPARENT COMPONENTS)

14. Summary and Conclusions

Verified by experiment and confirmed by both direct observation and indirect induction, the theory of ship motion under range action evolved in this paper would seem to be acceptable as an approximation at least to the true motion.

In terms of the theory a moored ship is treated as a mass in motion under the action of the periodic disturbing force of a seiche, but subject to the constraints of rope springs. The latter, unlike the simple mechanical springs of Hooke's law, are governed by an exponential tension-displacement relationship. The value of the numerical exponent in this relationship, together with the magnitude of the sea disturbance and the position of the ship, is the fundamental criterion which determines the condition under which the ship will resonate with the seiche in longitudinal motion. The mass of the ship itself usually has only a minor influence on the resonance condition, and ships both large and small, provided they are similarly tied and similarly situated, react in much the same way. Only when a variation in mass involves a large variation in deck (or mooring) height, does the ship size assume importance.

The periodicities of seiches in harbors are largely invariable, being dependent essentially upon the shape and the dimensions of the basins. Only those seiches which have a periodicity of about 2 min and less are capable of causing serious resonant motion in ships as normally moored. In particular, for most ships of commercial sizes (say, from 5,000 tons displacement to 30,000 tons displacement) the dangerous periodicities would seem to be in the range from 1 min down to about 20 sec. Very large vessels, whose deck height is considerable, are inclined to be affected more by periodicities between 1 min and 2 min. Low lying vessels, on the other hand, can oscillate dangerously in high frequency seiches of small amplitude, with periodicities of less than 20 sec.

The tighter the moorings of a ship are initially, the lower are the periodicities at which it can resonate. If ships could be tied with perfectly taut ropes, they would be completely unresponsive to all seiches, no matter how violent, with periodicities greater than about 20 sec. Provided, therefore, that a harbor is impervious to high frequency wave trains (periodicities of 30 sec and less), tight mooring of ships is an effective antidote to longitudinal range action.

When ships are poorly tied, or ropes have slackened considerably on the ebb tide, resonance is possible at almost any periodicity, and the longer period seiches (greater than 2 min) can become very dangerous. Since seiches of longer period in a harbor cannot in general be eliminated or subdued by any method short of entrance locks or gates, it is therefore important that reasonable tightness in moorings be maintained at all times.

In the transverse motion of a ship resonance is not strictly possible. A ship lying at the loop end of a transverse seiche is subjected to a comparatively small lateral displacement which cannot usually exceed about 1 ft, either way from rest position, under the worst conditions, and cannot call into effective play the spring action of the ropes. Nevertheless, this amount of movement is sufficient to occasion very large impact forces between the ship and the quay. The most dangerous periodicities in this action are again in the range from about 1 min

down to 20 sec. Of greatest importance is ship size, since the force of impact increases virtually as the square of its mass. Berths located along the longer side of a dock will probably always be more susceptible to the troubles inherent in transverse ranging than berths along the shorter side.

Transverse impact is undoubtedly the most serious feature of range action. Initially tight ropes will not be of much assistance in this case, since the rope tension that can be called into play to prevent lateral movement is extremely limited. If the evil is to be endured, the possible protective measures are the use of shock-absorbing fenders or of bow and stern anchors to hold ships off the quays.

Although it is possible to adopt palliative measures to enable ships to outride range action, the troubles and discomfort inherent in ship surging remain. It is eminently more satisfactory to inhibit the development of the sea disturbances themselves, and this, as this paper seeks to prove, only requires that a harbor should be made reasonably immune to penetration from short-period disturbances of 2 min and less. Oscillations of these frequencies are amenable to treatment by throttling through a chain system of basins, and herein lies the most effective solution of the range problem.

15. Acknowledgment

Grateful acknowledgment is due to W. C. Q. Joosting who assisted the writer at Capetown in securing and plotting measurements on ship motion and in subsequently searching the records for harmonic components. The writer desires also to express his appreciation to the Administration of the South African Railways and Harbours for permission to publish this paper.